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Residue Number System Fade Mitigation Technique with Error Detection and Correction on a Satellite Communication Link

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Abstract: Rain fade is the loss of signal power at the receiver of a telecommunication system mainly due to absorption and scattering caused by rain in the transmission medium, especially at frequencies above 10 GHz. In order to combat the loss of the signal power at the receiver, there is the need to employ rain fade mitigation techniques. Consequently, researchers have been studying how rain affects the signal in different geographical locations as well as proposing some mitigation techniques. Power control is one of the mitigation techniques that have been proposed. But this technique has some associated challenges. Increasing the power will lead to an increase in cost of transmission which will eventually be passed on to the consumer thereby making satellite services expensive. It requires high power in uplink and downlink which increases the burden either on user terminal or satellite payload. Also, because of health concerns there is a limit to the amount of power that can be radiated to the ground and this is governed by international agreements. Another power management drawback in using this technique is that, uplink power control is not efficient in directing the added power to only the ground station experiencing path attenuation, because the additional power is distributed to all locations within the satellite antenna coverage area. In this paper, we address the power control challenges, by leveraging on the inherent properties of Residue Number System (RNS) and Redundant Residue Number System (RRNS) to propose an RNS architecture using the moduli set $\{2^{2n+1}-1, 2^{2n}-1, 2^{2n}, 2^{4n+1}-1, 2^{2n}+1\}$ that can mitigate rain fade in the satellite link as well as detect and correct multiple errors. In digital communication systems, the bit energy, e_b, is the most important parameter in determining the communications link performance. Numerical analysis shows that the proposed scheme performs better than the traditional method as indicated in the high energy per bit value obtained in the proposed system in comparison with the traditional method, all other things being equal.

Keywords: Rain Fade Mitigation, Power Control, Residue Number System, Redundant Residue Number System

1. Introduction

The immense strength of satellite broadcasting lies in its ability to access a limitless number of sites without the need for any physical links irrespective of their location. A satellite receives the up-linked signal, lowers its frequency and rebroadcasts it to any geographical area desired [1]. The satellite signal transmission process is shown in Figure 1.

Rainfall is known to be the major cause of signal impairment at frequencies above 10 GHz [2]. In view of this, researchers have been proposing rain fade mitigation techniques. A research was conducted to mitigate rain fade using frequency diversity method by [3] in Malaysia. The frequency diversity method was used; however, this technique is not suitable because ground stations and satellites using this technique must be equipped to operate in dual frequency mode. The method is also complex because the receiver will have to pick up all the different signals. Power control is one of the mitigation techniques that have been proposed. In 2019, [4] conducted a research on rain attenuation mitigation on wireless communication link using adaptive power control. They concluded that Proportional

Integral Derivative has an enhanced response because it has shorter rising time and setting time when compared with proportional and proportional Integral controller systems. Their solution also focused on 5 GHz spectrum. But the use of power control is ineffective and expensive. This is because a satellite transmitter that offers coverage to a variety of users at different geographic location needs to work continuously at or near its peak power to overcome the overall attenuation encountered by only one of the ground stations. Again there are concerns regarding the safety of the amount of power that can be radiated to the ground and this is governed by international agreements [5]. Moreover, there are issues of intersystem interference with the increase in power level. This technique also requires some form of user intervention. Therefore it is imperative that other methods be considered to combat the effect of rain fade and restore acceptable performance on the communication link other than increasing the power. In this research, we leverage on the inherent properties of RNS in developing a cost effective solution to mitigate the effect of rain attenuation on the satellite communication link. We still maintain the existing encoding and decoding schemes. But at the physical layer of the computer architecture, we infuse RNS. This is achieved by using converters that changes the number system to RNS architecture before transmission. This is done using a forward converter at the transmitter. A reverse converter at the receiver then converts back from RNS to the traditional number representation. By this procedure, the energy per bit can be greatly enhanced so that even in the unlikely event of the signal encountering a rain event on the link, there will still be sufficient energy at the receiver to allow for the proper decoding of data at an acceptable bit error rate (BER). And in case an error occurs during transmission, we invoke the RRNS to detect and correct it at the receiver.

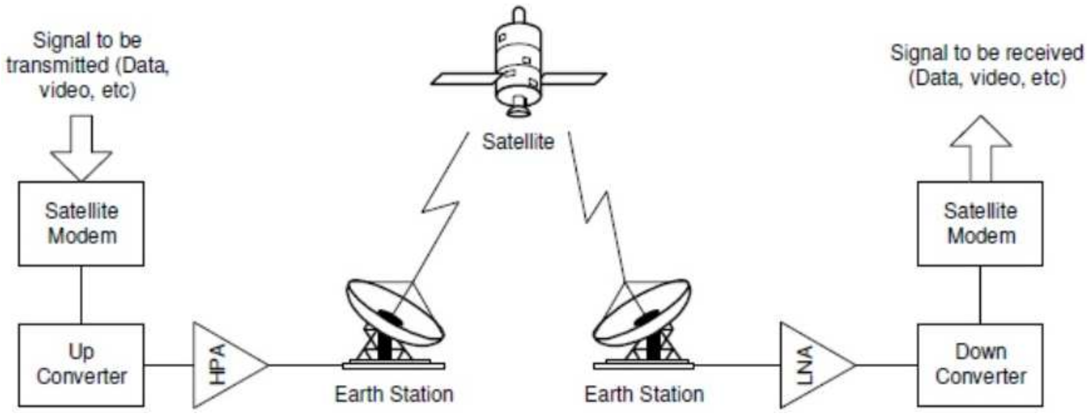

Figure 1. *The Satellite Transmission System.*

2. Residue Number System

Residue Number System (RNS) is a number system with numerous advantages. It is a well-established fact, that a number of digital devices naturally depend on number systems and to a large extend digital devices are built using binary number system. RNS has been used efficiently in communication systems [6], Digital Signal Processing for filtering, Convolutions and Correlations [7]. Residue Number System (RNS) is defined by the set S which includes N integers that are pair-wise relatively prime. That is S=$\{m_{1,} m_{2,} \ldots m_N\}$, where gcd $(m_i, m_j) = 1$ for $i, j = 1, \ldots N$ and $i, j \neq 1$ and gcd means the greatest common divisor [8].

Every integer X in $[0, M-1]$ can be uniquely represented with an N-tuple where,

$$M = \prod_{i=1}^{n} m_i, X \to (R_1, R_2, \ldots\ldots\ldots R_N) \; and \; R_i = |X|_{m_i} = (X \; mod \; m_i); for \; i = 1 \; to \; N.$$

The set S and the number R_i are called the moduli set and the residue of X modulo m_i, respectively [9-11].

Over the past years there has been renewed interest especially in the area of arithmetic computation and signal processing applications such as Fast Fourier transforms, digital filtering and image processing [12-15]. The inherent carry free operations, parallelism, borrow-free subtraction, single step multiplication without partial product and fault-tolerance properties of Residue Number System have made it a choice of technology for high precision and high throughput rate Digital Signal Processing applications where only repeated multiplications and additions are required [16 - 18]. Thus the desire for fast arithmetic computations and fault tolerant systems has made RNS a reliable choice.

3. Redundant Residue Number System

A Redundant Residue Number System (RRNS) is defined as a chosen RNS with additional redundant moduli. Each redundant modulus is generally greater than any of the moduli of the chosen moduli set. Assuming the standard RNS consists of the moduli set of $\{m_1, m_2, m_k\}$, the corresponding RRNS consists of a moduli set of $\{m_1, m_2, m_k, m_{k+2r}\}$ (r ≥ 1) [19- 21].

The RRNS has capability for error detection and correction. By using 2r (r ≥ 1) redundant moduli, r errors can be detected and corrected [19]. The residues in RNS serve as multiple data communication channels (fault tolerant; every digit result is completely independent) [22].

4. The Proposed Design

The complexity and varied problems associated with existing rain fade mitigation techniques makes it necessary to explore other methods. One way is the reduction in the data rate. But instead of reducing the size of data to be transmitted or transmitting one piece at a time, a better approach to use is by employing a different number system. The RNS is the most reliable solution, since this number system is able to reduce a given decimal to residues in respect to a given moduli set. And this was implemented by [23]. A plug in is designed to interface with the existing architecture just at the point where the data will be converted to binary. The data in the form of Universal Character Encoding Standard (UNICODE) values will be converted to residues. The residues to be transmitted will amount to fewer bits than the number of bits produced by the conventional number system. Therefore designing converters that can greatly reduce the number of bits before transmission will result in a higher energy-per-bit value at the transmitter before transmission. This will lead to a higher carrier-to-noise ratio at the receiver leading to better performance of the communication link, all other things being equal. However, there is the need for errors that might occur during transmission to be detected and corrected. The proposed system architecture is shown in Figure 2.

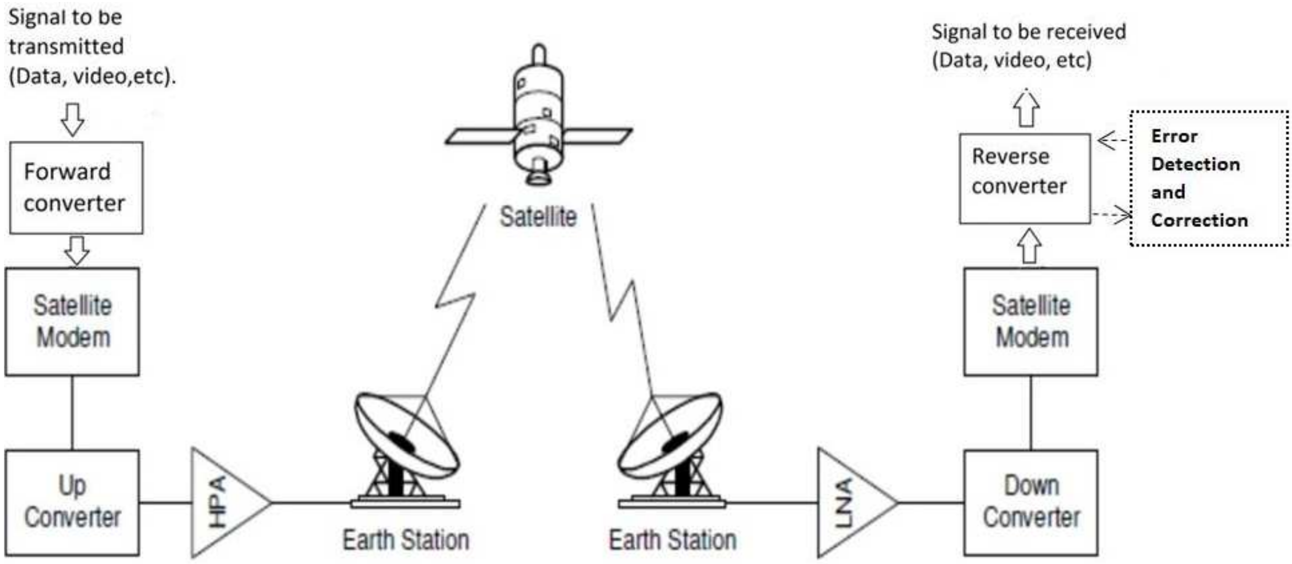

Figure 2. *Satellite communication link with RNS converters and Error detection and Correction mechanism.*

A. Forward Conversion Process For Moduli Set

$$\{2^{2n+1}-1, 2^{2n}-1, 2^{2n}, 2^{4n+1}-1, 2^{2n}+1\}$$

This moduli set was chosen because it gives a large dynamic range for small values of n. In satellite communication it is advisable to transmit large amount of data at a time because of the long delay. The moduli set of the proposed scheme works for both odd and even numbers of n > 1.

Given the moduli set, let $m_1 = 2^{2n+1}-1$, $m_2 = 2^{2n}-1$, $m_3 = 2^{2n}$; $m_4 = 2^{4n+1}-1$ and $m_5 = 2^{2n}+1$.

Where m_1, m_2, m_3 represent the information moduli and m_4, m_5 are the redundant moduli.

In general m_4, should be less than m_5. However because m_4 is extremely large, m_5 needs to be small so that the performance of the channel is not derailed because of the +1. But it can be seen that overall, the dynamic range of the redundant part is (6n + 2) which is greater than the information part which is (6n + 1).

An integer X in the range $[0, M)$ is a $(6n+1)$ bit number whose binary representation is given as follows:

$$X = X_{6n}X_{6n-}\dots X_1X_0 \tag{1}$$

This weighted representation has a unique equivalent RNS representation $x_i = |X|_{m_i} \Leftrightarrow (x_1, x_2, x_3)$ for the information part; and in order to compute the x_i 's, Equation (1) is partitioned into two $2n$-bit blocks and a $(2n+1)$-bit block as:

$$\left.\begin{aligned} \Lambda_1 &= \textstyle\sum_{j=0}^{2n-1} x_j 2^j \\ \Lambda_2 &= \textstyle\sum_{j=2n}^{4n-1} x_j 2^{j-2n} \\ \Lambda_3 &= \textstyle\sum_{j=4n}^{6n} x_j 2^{j-4n} \end{aligned}\right\} \tag{2}$$

this implies

$$X = \Lambda_1 + 2^{2n}\Lambda_2 + 2^{4n}\Lambda_3 \tag{3}$$

Such that,

$$\begin{aligned} x_1 &= |X|_{2^{2n+1}-1} \\ &= |\Lambda_1 + 2^{2n}\Lambda_2 + 2^{4n}\Lambda_3|_{2^{2n+1}-1} \\ &= ||\Lambda_1|_{2^{2n+1}-1} + |2^{2n}\Lambda_2|_{2^{2n+1}-1} + |2^{4n}\Lambda_3|_{2^{2n+1}-1}|_{2^{2n+1}-1} \\ &= |\Lambda_1 + 2^{2n}\Lambda_2 + 2^{2n-1}\Lambda_3|_{2^{2n+1}} \end{aligned} \tag{4}$$

$$x_2 = |X|_{2^{2n}-1} = |\Lambda_1 + 2^{2n}\Lambda_2 + 2^{4n}\Lambda_3|_{2^{2n}-1}$$

$$=\left||\Lambda_1|_{2^{2n}-1} + |2^{2n}\Lambda_2|_{2^{2n}-1} + |2^{4n}\Lambda_3|_{2^{2n}-1}\right|_{2^{2n}-1}$$

$$= |\Lambda_1 + \Lambda_2 + \Lambda_3|_{2^{2n}-1} \quad (5)$$

and

$$x_3 = |X|_{2^{2n}} = \Lambda_1 \quad (6)$$

For the information part while the redundant part is:

$$x_4 = |X|_{2^{4n+1}-1} = |\Lambda_1 + 2^{2n}\Lambda_2 + 2^{4n}\Lambda_3|_{2^{4n+1}-1}$$

$$=\left||\Lambda_1|_{2^{4n+1}-1} + |2^{2n}\Lambda_2|_{2^{4n+1}-1} + |2^{4n}\Lambda_3|_{2^{4n+1}-1}\right|_{2^{4n+1}-1}$$

$$= |\Lambda_1 + 2^{2n}\Lambda_2 + 2^{4n}\Lambda_3|_{2^{4n+1}-1} \quad (7)$$

$$x_5 = |X|_{2^{2n}+1} = |\Lambda_1 + 2^{2n}\Lambda_2 + 2^{4n}\Lambda_3|_{2^{2n}+1}$$

$$= \left||\Lambda_1|_{2^{2n}+1} + |2^{2n}\Lambda_2|_{2^{2n}+1} + |2^{4n}\Lambda_3|_{2^{2n}+1}\right|_{2^{2n}+1}$$

$$= |\Lambda_1 - \Lambda_2 + \Lambda_3|_{2^{2n}+1} \quad (8)$$

The block diagram for the forward conversion of the information part is represented in Figure 3; it employs simple adders made up of Carry Save Adders (CSAs) and Carry Propagate Adders (CPAs). The block diagram for the forward conversion for the redundant part is shown in Figure 4.

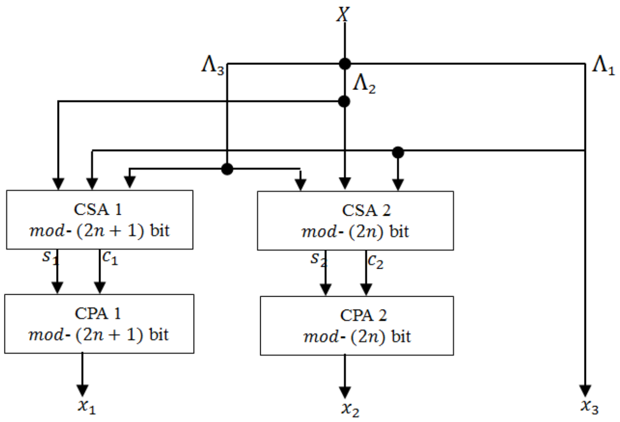

Figure 3. *Block Diagram of Forward Convertor for Information part.*

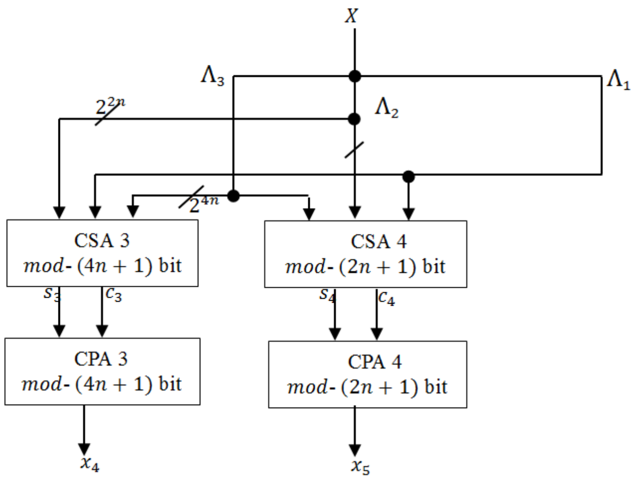

Figure 4. *Block Diagram of Forward Convertor for Redundant part.*

B. Bit Energy Computation

For digital communications, the bit energy, e_b , is more helpful for determining the link's performance than the carrier power. The link efficiency can be reduced in two ways: if the carrier power, c, is decreased, and/or if the noise power is increased. For example, the carrier power is reduced by the increase in the number of bits to be transmitted and the noise power is increased by the absorption and scattering of the signal by rain drops. Thus the larger the value of e_b, the better the performance of the link holding the noise power constant. By employing RNS architecture we will greatly reduce the number of bits to be transmitted thereby allowing more power to be available to fewer bits. In Equation (9), a relationship is established between the bit energy and the carrier power as

$$e_b = c\,T_b \quad (9)$$

where T_b is the bit duration in second (s) and c is the carrier power in watts. The energy-per-bit to noise density ratio, $\left(\frac{e_b}{n_o}\right)$, is the most widely used parameter to evaluate the performance of a digital communication link. And $\left(\frac{e_b}{n_o}\right)$ is related to $\left(\frac{c}{n_o}\right)$ by

$$\left(\frac{e_b}{n_o}\right) = T_b\left(\frac{c}{n_o}\right) = \frac{1}{R_b}\left(\frac{c}{n_o}\right) \quad (10)$$

Therefore by eliminating the noise component and power, it can be observed from Equation (9) that

$$e_b \propto \frac{1}{R_b} \quad (11)$$

Again, introducing the carrier power, c, as a constant, Equation (11) can be rewritten as

$$e_b = \frac{c}{R_b} \quad (12)$$

where e_b is the energy-per-bit, c is the power at the carrier and R_b is the bit rate, in bits per second (bps).

C. Reverse Conversion Process

The Mixed Radix Conversion (MRC) is employed to decode any number X in RNS representation to its binary/decimal equivalent for the information part. MRC was chosen because it suits the moduli set well especially with regards to computing the multiplicative inverses. It also avoids the use of big M which can result in complex computations. Also, the sequential nature of the MRC allows for a simplified architecture.

The general form of the MRC is given as follows;

$$X = d_1 + d_2m_1 + d_3m_1m_2 + \cdots + d_nm_1m_2m_3 \ldots m_{n-1} \quad (13)$$

Where $d_i, i = 1,2,\ldots,n$ are the Mixed Radix Digits (MRDs) and computed as follows:

$$d_1 = x_1,$$

$$d_2 = \left|(x_2 - d_1)|m_1^{-1}|_{m_2}\right|_{m_2},$$

$$d_3 = \left|\left((x_3 - d_1)|m_1^{-1}|_{m_3} - d_2\right)|m_2^{-1}|_{m_3}\right|_{m_3},$$

$$\vdots$$

$$d_n = \left|\left(\dots\left((x_3 - d_1)|m_1^{-1}|_{m_n} - d_2\right)|m_2^{-1}|_{m_n} - \dots - d_{n-1}\right)|m_{n-1}^{-1}|_{m_n}\right|_{m_n} \quad (14)$$

Therefore, X in the interval $[0, M)$ can be uniquely represented. And so we can re-write Equation (14) as

$$d_1 = x_1,$$

$$d_2 = |(x_2 - d_1)(1)|_{2^{2n}-1} = |x_2 - x_1|_{2^{2n}-1},$$

and

$$d_3 = \left|\left((x_3 - d_1)(-1) - d_2\right)(-1)\right|_{2^{2n}} = |(x_3 - x_1 + d_2|_{2^{2n}} \quad (15)$$

And Equation (13) then becomes

$$X = x_1 + d_2(2^{2n+1} - 1) + d_3(2^{2n+1} - 1)(2^{2n} - 1)$$

$$= x_1 + 2^{2n+1}d_2 - d_2 + 2^{4n+1}d_3 - 2^{2n}d_3 - 2^{2n+1}d_3 + d_3 \quad (16)$$

For the redundant part, the reverse conversion process follows the same processes as that of the above equations, which deals with only the information part; but in case there is an error in the transmission and the two redundant moduli need to be employed, then we present a reverse converter just for the redundant part, which can be employed in the detection and correction of the possible errors.

Given the redundant moduli, m_R as $m_4 = 2^{4n+1} - 1$ and $m_5 = 2^{2n} + 1$; we employ the MRC in Equation (13) in order to achieve the reverse conversion process. Thus, we need to find a_1, a_2 and $|m_4^{-1}|_{m_5}$ to compute the value of the redundant part as follows;

$$X_R = a_1 + (2^{4n+1} - 1)a_2$$

$$= x_4 + 2^{4n+1}a_2 - a_2 \quad (17)$$

Since,

$$a_1 = x_4 \quad (18)$$

$$a_2 = \left|(x_5 - x_4)|m_4^{-1}|_{m_5}\right|_{m_5} \quad (19)$$

D. Hardware Realization

We now simplify Equations (15) and (16) in binary as follows;

$$d_1 = x_{1,2n}x_{1,2n-1} \dots\dots\dots x_{1,1}x_{1,0} \quad (20)$$

$$d_2 = \left|\underbrace{x_{2,2n-1}x_{2,2n-2} \dots\dots . x_{2,1}x_{2,0}}_{2n-bits} - \left|\underbrace{x_{1,2n}x_{1,2n-1} \dots\dots\dots x_{1,1}x_{1,0}}_{2n+1-bits}\right|_{2^{2n}-1}\right|_{2^{2n}-1}$$

$$= \left|\underbrace{x_{2,2n-1}x_{2,2n-2} \dots\dots . x_{2,1}x_{2,0}}_{2n-bits} + \underbrace{x_{11,2n-1}x_{11,2n-2} \dots\dots\dots x_{11,1}x_{11,0}}_{2n-bits}\right|_{2^{2n}-1}$$

$$= \underbrace{d_{2,2n-1}d_{2,2n-2} \dots\dots\dots\dots . d_{2,1}d_{2,0}}_{2n-bits} \quad (21)$$

Where, $x_{11} = |-x_1|_{2^{2n}-1}$

and

$$d_3 = \left|\underbrace{x_{3,2n-1} \dots\dots . x_{3,1}x_{3,0}}_{2n-bits}\,\underbrace{x_{11,2n-1} \dots x_{11,1}x_{11,0}}_{2n-bits} + \underbrace{d_{2,2n-1} \dots\dots . d_{2,1}d_{2,0}}_{2n-bits}\right|_{2^{2n}}$$

$$= \underbrace{d_{3,2n-1}d_{3,2n-2} \dots\dots . d_{3,1}d_{3,0}}_{2n-bits} \quad (22)$$

The binary representation and computations for the redundant part are as follows:

$$a_1 = x_{4,4n}x_{4,4n-1} \dots x_{4,1}x_{4,0} \quad (23)$$

$$a_2 = \left|\underbrace{x_{5,2n}x_{5,2n-1} \dots x_{5,1}x_{5,0}}_{2n+1} - \left(\underbrace{x_{4,4n}x_{4,4n-1} \dots x_{4,1}x_{4,0}}_{4n+1}\right)\right|_{2^{2n}+1}$$

$$= \left|\underbrace{x_{5,2n}x_{5,2n-1} \dots x_{5,1}x_{5,0}}_{2n+1} + \left|\bar{x}_{4,4n}\bar{x}_{4,4n-1} \dots \bar{x}_{4,1}\bar{x}_{4,0}\right|_{2^{2n}+1}\right|_{2^{2n}+1}$$

$$= a_{2,2n}a_{2,2n-1} \dots a_{2,1}a_{2,0} \quad (24)$$

And finally,

$$X_R = \underbrace{x_{4,4n}x_{4,4n-1} \dots x_{4,1}x_{4,0}}_{4n+1} + 2^{4n+1}\left(a_{2,2n}a_{2,2n-1} \dots a_{2,1}a_{2,0}\right) - \left(a_{2,2n}a_{2,2n-1} \dots a_{2,1}a_{2,0}\right)$$

$$= \overbrace{00 \dots 0}^{2n+1}\underbrace{x_{4,4n} \dots x_{4,1}x_{4,0}}_{4n+1} + a_{2,2n} \dots a_{2,1}a_{2,0}\overbrace{00 \dots 0}^{4n+1} + \overbrace{11 \dots 1}^{4n+1}\bar{a}_{2,2n} \dots \bar{a}_{2,1}\bar{a}_{2,0} \quad (25)$$

A simplification of Equation (16) for implementation is as follows:

$$X = L_1 + L_2 + L_3 + L_4 = \underbrace{\begin{matrix} \overbrace{0 \dots 0}^{2n}\underbrace{L_{1,4n} \dots L_{1,1}L_{1,0}}_{4n+1} + \overbrace{0 \dots 0}^{2n}\underbrace{L_{2,4n} \dots L_{2,1}L_{2,0}}_{4n+1} + \overbrace{0 \dots 0}^{2n+1}\underbrace{L_{3,4n-1} \dots L_{3,1}L_{3,0}}_{4n} \\ + \\ \underbrace{L_{4,6n} \dots L_{4,1}L_{4,0}}_{6n+1} \end{matrix}}_{6n+1} \tag{26}$$

where,

$$L_1 = A - d_2 + d_3 = \underbrace{A_{4n}A_{4n-1} \dots \dots . A_1A_0}_{4n+1} + \overbrace{11 \dots 1}^{2n+1}\underbrace{\bar{d}_{2,2n-1} \dots \bar{d}_{2,1}\bar{d}_{2,0}}_{2n-bits} + \underbrace{d_{3,2n-1} \dots d_{3,1}d_{3,0}}_{2n-bits}\overbrace{00 \dots 0}^{2n+1} \tag{27}$$

and

$$A = x_1 + 2^{2n+1}d_2 = \underbrace{x_{1,2n}x_{1,2n-1} \dots x_{1,1}x_{1,0}}_{2n+1-bits}\overbrace{0 \dots .00}^{2n-bits} \bowtie \underbrace{d_{2,2n-1}d_{2,2n-2} \dots d_{2,1}d_{2,0}}_{2n-bits}\overbrace{0 \dots .00}^{2n+1-bits}$$

$$= \underbrace{x_{1,2n}x_{1,2n-1} \dots x_{1,1}x_{1,0}d_{2,2n-1}d_{2,2n-2} \dots d_{2,1}d_{2,0}}_{4n+1} = A_{4n}A_{4n-1} \dots A_1A_0 \tag{28}$$

Whereas

$$L_2 = -2^{2n+1}d_3 = \underbrace{\underbrace{\bar{d}_{3,2n-1}\bar{d}_{3,2n-2} \dots \bar{d}_{3,1}\bar{d}_{3,0}}_{2n-bits}\overbrace{11 \dots 1}^{2n+1}}_{4n+1} \tag{29}$$

$$L_3 = -2^{2n}d_3 = \underbrace{\underbrace{\bar{d}_{3,2n-1}\bar{d}_{3,2n-2} \dots \bar{d}_{3,1}\bar{d}_{3,0}}_{2n-bits}\overbrace{11 \dots 1}^{2n}}_{4n} \tag{30}$$

$$L_4 = 2^{4n+1}d_3 = \underbrace{\underbrace{d_{3,2n-1}d_{3,2n-2} \dots d_{3,1}d_{3,0}}_{2n-bits}\overbrace{00 \dots 0}^{4n+1}}_{6n+1} \tag{31}$$

The schematic diagram for the reverse conversion process is shown in Figure 5 for the information part and Figure 6 for the redundant part. The anticipated implementation of the scheme is based on simple CSAs and CPAs. It begins with an operands preparation unit (OPU) which prepares and manipulates the routing of the bits of the respective residues once inputted. The second MRD, d_2 is computed using CPA1 which is $2n$-bits wide; the concatenation of bits does not require a hardware unit, but its results is relevant for further processing. CSA1 and CPA2 are used to compute for the third MRD, d_3 after which, the rest of the addition process is done in a cascading fashion thereby reducing the complexity which results in a simplified architecture. CPA 3 computes the save and carry from CSA 5 in order to get back the decimal or binary number, X. Regarding the hardware requirements of the unit; the total hardware (area) requirement is $(30n + 5)$ bits of full adders and a delay imposition of $(10n + 6)$ bits of full adders. For the redundant part, the converter is made up of only three simple adders, two CPA's and one CSA.

5. Numerical Illustration

With regards to the information part, given the moduli set $\{2^{2n+1} - 1,\ 2^{2n} - 1,\ 2^{2n}\}$, take $n = 2$. Consider X representing a character of a message to be transmitted, whose integer value (i.e. UNICODE) is given as $X = 6892$. Then the conversion process is as follows; $6892 = 1101011101100$ (13-bits, since X is a $(6n + 1)$-bit number)

Thus, $\Lambda_1 = 1100, \Lambda_2 = 1110$ and $\Lambda_3 = 11010$

Therefore;

$$|6892|_{2^5-1} = |6892|_{31} = |12 + |16(14)|_{31} + |8(26)|_{31}|_{31} = |12 + 7 + 22|_{31} = |41|_{31} = 10,$$

$$|6892|_{2^4-1} = |6892|_{15} = |12 + 14 + |26|_{15}|_{15} = |12 + 14 + 11|_{15} = |37|_{15} = 7$$

And $|6892|_{2^4} = |6892|_{16} = 12$

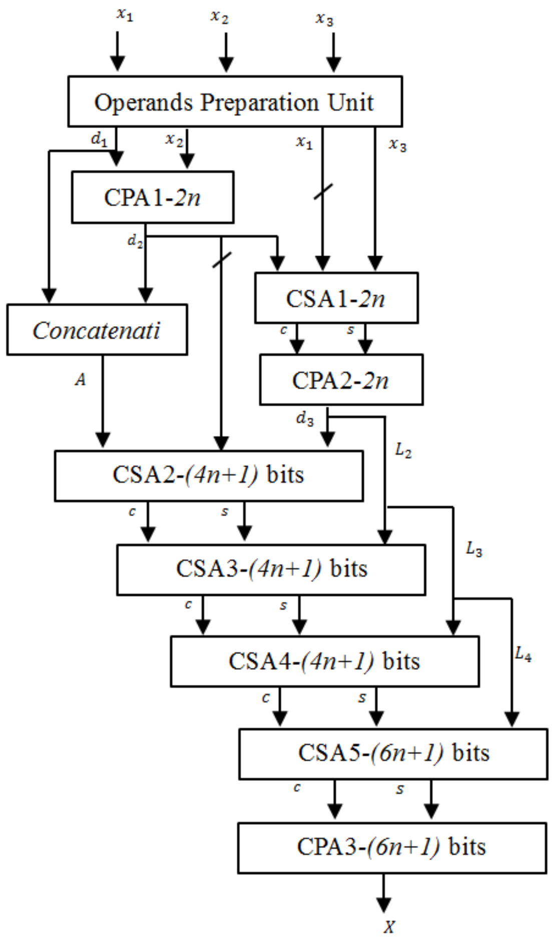

Figure 5. *Schematic Diagram of Reverse Conversion Process for Information part.*

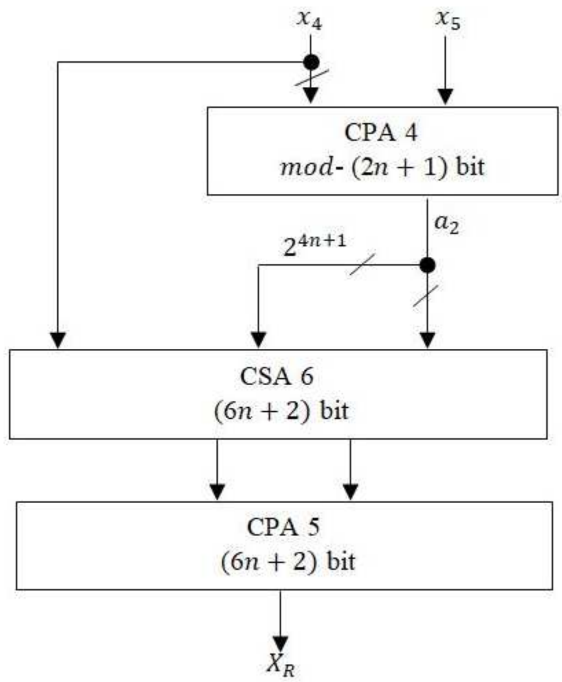

Figure 6. *Schematic Diagram of Reverse Conversion Process for Redundant part.*

And so $|6892|_{31|15|16} = (10,7,12)_{31|15|16}$

10 = 01010 7 = 0111 12 = 1100

And so $|6892|_{31|15|16} = (10,7,12)_{31|15|16}$

From the above illustration, it can be seen that the traditional method will convert the UNICODE message, X = 6892, to binary which is $6892 = 1101011101100$ (13-bits, since X is a $(6n+1)$-bit number). But this representation has a unique equivalent in RNS which is $|6892|_{31|15|16} = (10,7,12)_{31|15|16}$. Each of the moduli sets are channels that can be used to transmit the message X, but it is the binary equivalent of the residues that will be transmitted. The binary equivalent of $(10,7,12)_{31|15|16}$ is as follows:

10 = 01010 7 = 0111 12 = 1100

In order to perform the reverse conversion we find the MRD's a_1, a_2 and a_3 and compute the MRC in order to get back the value of X (i.e. message transmitted):

But $a_1 = x_1 = 10$

$$a_2 = |7 - 10|_{15} = 12$$

$$a_3 = \left|\left((12 - 10) \times 5 - 12\right) \times 15\right|_{16} = 14$$

Therefore,

$$X = 10 + 12(31) + 14(31)(15) = 6892$$

Thus for $X = 6892$, the transmitted message in RNS was $|6892|_{31|15|16} = (10,7,12)_{31|15|16}$ and the corresponding redundant residues are $(249,7)_{511|17}$.

Now, let us assume x_1 is transmitted in error and the received message is $\{\underline{15},7,12\}$. This can be detected and corrected by first of all computing the wrong integer value at the receiver as follows;

$$a_1 = x_1 = 15$$

$$a_2 = |7 - 15|_{15} = 7$$

$$a_3 = \left|\left((12 - 15) \times 15 - 7\right) \times 15\right|_{16} = 4$$

and

$$X = 15 + 7(31) + 4(31)(15) = 2092$$

For the detecting if there is an error, the fourth modulus (first redundant modulus) is utilized. A modulo operation is performed on the wrong integer value computed at the receiver using the fourth modulus and compared with the value of the fourth residue. And it is expected that both will result in the same value. That is, $|2092|_{2^{4n+1}-1} = |2092|_{511} = 48$, However $48 \neq 249$ as expected. It is therefore concluded that an error has occurred in the transmission of the message.

For the correction process, the fifth modulus of the redundant moduli set is invoked. The original message is reconstructed using three moduli out of the five with their corresponding residues and MRC reverse converter. There are ten possible conditions for selecting three moduli out of five moduli $\left(i.e\ \binom{5}{3} = 10\right)$.

The combinations are as follows;

$\{m_1, m_2, m_3\}$ with corresponding residue vector $\{x_1, x_2, x_3\}$
$\{m_1, m_2, m_4\}$ with corresponding residue vector $\{x_1, x_2, x_4\}$
$\{m_1, m_2, m_5\}$ with corresponding residue vector $\{x_1, x_2, x_5\}$
$\{m_1, m_3, m_4\}$ with corresponding residue vector $\{x_1, x_3, x_4\}$
$\{m_1, m_3, m_5\}$ with corresponding residue vector $\{x_1, x_3, x_5\}$
$\{m_1, m_4, m_5\}$ with corresponding residue vector $\{x_1, x_4, x_5\}$
$\{m_2, m_3, m_4\}$ with corresponding residue vector $\{x_2, x_3, x_4\}$
$\{m_2, m_3, m_5\}$ with corresponding residue vector $\{x_2, x_3, x_5\}$
$\{m_2, m_4, m_5\}$ with corresponding residue vector $\{x_2, x_4, x_5\}$
$\{m_3, m_4, m_5\}$ with corresponding residue vector $\{x_3, x_4, x_5\}$

For $\{m_1, m_2, m_4\}$, the moduli set is $\{31, 15, 511\}$ and the respective multiplicative inverses are $|m_1^{-1}|_{m_2} = 1, |m_1^{-1}|_{m_4} = 33, |m_2^{-1}|_{m_4} = 477$ and thus results in $X_{124} = 6892$

For $\{m_1, m_2, m_5\}$, the moduli set is $\{31, 15, 17\}$;

$$|m_1^{-1}|_{m_2} = 1,\ |m_1^{-1}|_{m_5} = 11,\ |m_2^{-1}|_{m_5} = 8,\ X_{125} = 6892$$

For $\{m_1, m_3, m_4\}$, the moduli set is $\{31,16,511\}$ and the respective multiplicative inverses are

$$|m_1^{-1}|_{m_3} = 15, |m_1^{-1}|_{m_4} = 33, |m_3^{-1}|_{m_4} = 32 \text{ and}$$

$$X_{134} = 6892$$

For $\{m_1, m_3, m_5\}$, the moduli set is $\{31, 16, 17\}$, the respective multiplicative inverses are

$$|m_1^{-1}|_{m_3} = 15,\ |m_1^{-1}|_{m_5} = 11,\ |m_3^{-1}|_{m_5} = 16 \text{ and}$$

$$X_{135} = 6892$$

For $\{m_1, m_4, m_5\}$, the moduli set is $\{31, 511, 17\}$, the respective multiplicative inverses are

$$|m_1^{-1}|_{m_4} = 33,\ |m_1^{-1}|_{m_5} = 11,\ |m_4^{-1}|_{m_5} = 1 \text{ and}$$

$$X_{145} = 6892$$

For $\{m_2, m_3, m_4\}$, the moduli set is $\{15, 16, 511\}$, the respective multiplicative inverses are

$$|m_2^{-1}|_{m_3} = 15, |m_2^{-1}|_{m_4} = 477, |m_3^{-1}|_{m_4} = 32, \text{ where}$$

$$X_{234} = 31420$$

For $\{m_2, m_3, m_5\}$, the moduli set is $\{15,16,17\}$, the respective multiplicative inverses will be $|m_2^{-1}|_{m_3} = 15, |m_2^{-1}|_{m_5} = 8,\ |m_3^{-1}|_{m_5} = 16$ and $X_{235} = 1180$

For $m_2, m_4, m_5\}$, the moduli set is $\{15, 511, 17\}$, the respective multiplicative inverses will be

$$|m_2^{-1}|_{m_4} = 477,\ |m_2^{-1}|_{m_5} = 8,\ |m_4^{-1}|_{m_5} = 1,$$

$$X_{245} = 85075$$

For $\{m_3, m_4, m_5\}$, the moduli set is $\{16, 511, 17\}$, the respective multiplicative inverses are

$$|m_3^{-1}|_{m_4} = 32, |m_3^{-1}|_{m_5} = 16, |m_4^{-1}|_{m_5} = 1, X_{345} = 24266$$

Therefore, the following are obtained;

$$\{x_1, x_2, x_3\} = \{15, 7,12\} \leftrightarrow X_{123} = 6892$$

$$\{x_1, x_2, x_4\} = \{15, 7,249\} \leftrightarrow X_{124} = 6892$$

$$\{x_1, x_2, x_5\} = \{15, 7,7\} \leftrightarrow X_{125} = 6892$$

$$\{x_1, x_3, x_4\} = \{15, 12,249\} \leftrightarrow X_{134} = 6892$$

$$\{x_1, x_3, x_5\} = \{15, 12,7\} \leftrightarrow X_{135} = 6892$$

$$\{x_1, x_4, x_5\} = \{15, 249,7\} \leftrightarrow X_{145} = 6892$$

$$\{x_2, x_3, x_4\} = \{7, 12,249\} \leftrightarrow X_{234} = 31420$$

$$\{x_2, x_3, x_5\} = \{7, 12,7\} \leftrightarrow X_{235} = 1180$$

$$\{x_2, x_4, x_5\} = \{7, 249,7\} \leftrightarrow X_{245} = 85075$$

$$\{x_3, x_4, x_5\} = \{12, 249,7\} \leftrightarrow X_{345} = 24266$$

From these results, we observe that every iteration for X mostly yields an illegitimate value except when the position where the error is introduced is included in the iteration. That is $X_{123}, X_{124}, X_{125}, X_{134}, X_{135}$ *and* X_{145} all result in one legitimate number. This sole legitimate number turns out in all cases to be the correct integer that was transmitted. Now to correct the residue in error, a modulo operation of the repeated number is done using the modulo corresponding to the channel in error as shown below:

$$|6892|_{2^5-1} = |6892|_{31} = |12 + |16(14)|_{31} + |8(26)|_{31}|_{31}$$
$$= |12 + 7 + 22|_{31} = |41|_{31} = 10.$$

6. Performance Analysis

The performance of the communication link is determined by the energy per bit value. We can compare the worst case scenario of RNS architecture with the traditional method in terms of computing the bit energy. Let's assume a carrier power of 1W at the transmitter, with the traditional method, the transmit energy per bit, $e_b = \frac{1}{13} = 0.07692307692$ Joules. In RNS representation, in the worst case scenario, $e_b = \frac{1}{5} = 0.2$ Joules. It can be seen that excess energy of 0.1230769231 Joules is available in RNS than in the traditional method. The RNS architecture therefore will perform better than the traditional method in the unlikely event of rainfall along the satellite communication path. In the second and third transmission channels, $e_b = \frac{1}{4} = 0.25$ Joules. Table 1 shows the variation in the energy per bit between the proposed scheme using RNS and the traditional method. Therefore when there is rain fade along the satellite link, all other things being equal the proposed scheme will have enough energy available at the receiver to allow for proper data decoding at an acceptable bit error rate. It is also interesting to note that even the energy difference between the proposed scheme and the traditional scheme is higher than that of the traditional scheme. This can be seen in Figure 7.

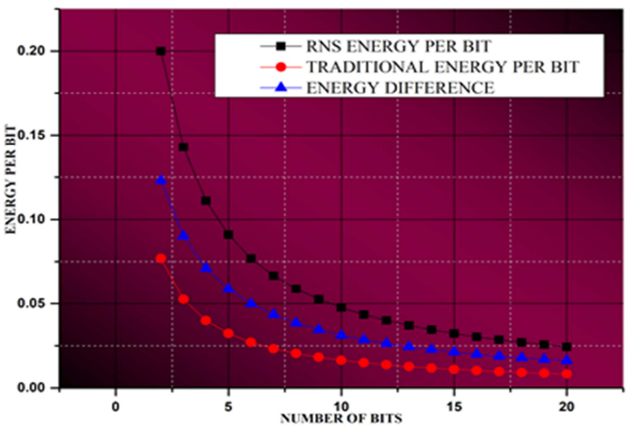

Figure 7. *Difference in Energy per Bit.*

Table 1. *Variation in Energy per Bit.*

N	RNS ENERGY PER BIT	TRADITIONAL ENERGY PER BIT	ENERGY DIFFERENCE
2	0.2	0.076923077	0.123076923
3	0.142857143	0.052631579	0.090225564
4	0.111111111	0.04	0.071111111
5	0.090909091	0.032258065	0.058651026
6	0.076923077	0.027027027	0.04989605
7	0.066666667	0.023255814	0.043410853
8	0.058823529	0.020408163	0.038415366
9	0.052631579	0.018181818	0.034449761
10	0.047619048	0.016393443	0.031225605
11	0.043478261	0.014925373	0.028552888
12	0.04	0.01369863	0.02630137
13	0.037037037	0.012658228	0.024378809
14	0.034482759	0.011764706	0.022718053
15	0.032258065	0.010989011	0.021269054
16	0.03030303	0.010309278	0.019993752
17	0.028571429	0.009708738	0.018862691
18	0.027027027	0.009174312	0.017852715
19	0.025641026	0.008695652	0.016945373
20	0.024390244	0.008264463	0.016125781

7. Conclusion and Future Work

A novel RNS fade mitigation technique has been proposed on the satellite communication link with error detection and correction capabilities. The technique involves the use of converters and a different number system from the traditional number system to reduce the number of bits to be transmitted. Forward converters to convert to RNS system before transmission and reverse converters to convert back to the traditional number system at the receiver. The moduli set chosen, $\{2^{2n+1}-1, 2^{2n}-1, 2^{2n}, 2^{4n+1}-1, 2^{2n}+1\}$, works for both odd and even numbers of n. Errors occurring in any of the channels can be detected and corrected. However the maximum number of errors that can be detected and corrected is 2n+1, occurring in a single channel. Therefore, if the hardware realization of the proposed architecture is incorporated into the satellite communication link, it will help mitigate the attenuation due to rain since more energy will be made available to the individual bits. In future, multiple channel error detection and correction scheme will be implemented.

Conflict of Interest

The authors declare no conflict of interest in this research work.

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